

ABSTRACT ALGEBRA
EXERCISE SHEET 15

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In this problem sheet we are going to show that a particular polynomial of degree 5 is not solvable. All points on this sheet are bonus points. We consider the polynomial $P := X^5 - 9X + 3$ over \mathbb{Q} .

Problem 1 (10 points*). Show that over \mathbb{Q} the polynomial P is irreducible.

Problem 2 (10 points*). Show that P has exactly 3 real and 2 non-real complex roots. The latter are complex conjugate to each other.

Problem 3 (10 points*). Let \mathbb{Q}_P be the splitting field of P in \mathbb{C} . Prove that $\text{Gal}(\mathbb{Q}_P|\mathbb{Q})$ is isomorphic to the 5th symmetric group \mathfrak{S}_5 .

Problem 4 (10 points*). Show that $[\mathfrak{S}_5, \mathfrak{S}_5] = A_5$ and $[A_5, A_5] = A_5$.