ABSTRACT ALGEBRA
EXERCISE SHEET 10

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Problem 1 (10 points)**.** Prove Proposition 103.

Problem 2 (10 points)**.** Let A be a UFD and $\mathrm{P} \in \mathrm{A}[X]$ be a non-zero polynomial.

(i) Show that the content of P, see Definition 99, is well-defined.
(ii) Prove Proposition 100.

Problem 3 (10 points)**.** Find the greatest common divisor of the following two polynomials.

$$X^6 + 3X^5 + 7X^4 + 12X^3 + 15X^2 + 9X + 9, \; X^4 + 6X^3 + 13X^2 + 12X + 3.$$

Problem 4 (10 points)**.** Consider the polynomials $\mathrm{P}(X_1, X_2, X_3) := X_1^2 + X_1^2 X_2 + X_1 X_2 X_3$ and

$$\mathrm{Q} := \Sigma_{\sigma \in \mathfrak{S}_3} \sigma(\mathrm{P}).,$$

all as polynomials in $\mathbb{Z}[X_1, X_2, X_3]$. Find a polynomial $\tilde{\mathrm{Q}} \in \mathbb{Z}[U_1, U_2, U_3]$ of weight 3 such that $\tilde{\mathrm{Q}}(S_1, S_2, S_3) = \mathrm{Q}$ where S_1, S_2, S_3 are the elementary symmetric polynomials in X_i, $i = 1, 2, 3$.

Date: Please hand in before the lecture by **06.05.2021**. For all exercises the results need to be proven.